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SHORTER COMMUNICATIONS

TEMPERATURE DISTRIBUTION IN FLOOR HEATING SYSTEMS

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NOMENCLATURE

T, temperature in the floor;
T_1, temperature of the heating pipe;
T_0, room temperature;
G, Green's function.

Greek symbols

α, thermal convection coefficient;
λ, thermal conduction coefficient;
ρ, source function along the boundary.

1. INTRODUCTION

Floor heating systems have been receiving more attention in recent years. Due to the rather large heating surface low temperatures can be used. From the architectural point of view, floor heating is very attractive as no radiator or similar heating devices are to be placed against walls. Floor heating also works in the same direction as the natural convection of heated air, so that room heating will be performed more efficiently. The thermal gradient is directed downwards, yielding a better thermal comfort.

The main advantage of floor heating is undoubtedly the low temperature of the circulating liquid due to the large floor area. This is very important as one wants to include solar energy for space heating as low temperatures can still be used. The incorporation of a heat pump is also more attractive as its gain will increase due to the lower temperature required by the floor heating installation.

A disadvantage of floor heating is that the maximum floor temperature should be limited (usually 26–29°C). Hence the maximum heat flux is limited. For well insulated houses, however, this limitation causes no problems.

In this paper the temperature in a heating floor and the thermal flux will be calculated. This will be done numerically by a boundary integral equation method, which is extremely suited for arbitrary geometries. This has the advantage that a change in the geometry (e.g. a variation in the distance between the heating coils) can be easily performed.

2. MODEL

Consider the structure shown in Fig. 1. The bottom layer is assumed to be a perfect insulator. The distance a between the heating pipes can be easily changed around the mean value $a = 15$ cm. The thickness d is also taken to be variable. The floor material is considered to be homogeneous from the thermal point of view (i.e. same λ), which is a reasonable assumption in actual cases.

Due to the periodicity of the structure shown on Fig. 1, one has only to consider one half period for the solution of the problem. The boundary conditions are (Fig. 2):

$$\nabla T \cdot \mathbf{u}_n = 0 \quad \text{on } BC, DE, EF \text{ and } FA \qquad (1)$$

$$\lambda \nabla T \cdot \mathbf{u}_n = \alpha(T - T_0) \quad \text{on } AB \qquad (2)$$

$$T = T_1 \quad \text{on } CD \qquad (3)$$

$\mathbf{u}_n$ denotes the normal unity vector; T is the temperature distribution in the floor; T_1 the temperature of the heating pipe and T_0 the room temperature; α is the thermal convection coefficient. The external surface of the heating pipe is taken to be isothermal, which certainly will be the case when stainless steel is used.

In order to solve the Laplace equation in the area S (with boundary ∂S) shown on Fig. 2, the following solution is proposed:

$$T(\mathbf{r}) = \oint_{\partial S} \rho(\mathbf{r}')G(\mathbf{r}|\mathbf{r}')\mathrm{d}C' \qquad (4)$$

where $\rho(\mathbf{r}')$ is still an unknown function along the boundary ∂S. $G(\mathbf{r}|\mathbf{r}')$ is the so-called Green's function of the Laplace equation:

$$G(\mathbf{r}|\mathbf{r}') = \frac{1}{2\pi}\ln|\mathbf{r}-\mathbf{r}'| \qquad (5)$$

where

$$\nabla^2 G(\mathbf{r}|\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}') \qquad (6)$$

One can easily verify that the proposed solution (4) will always satisfy the Laplace equation. In order to determine the

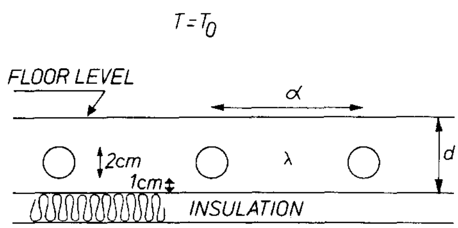

Fig. 1. Cross section of floor heating systems.

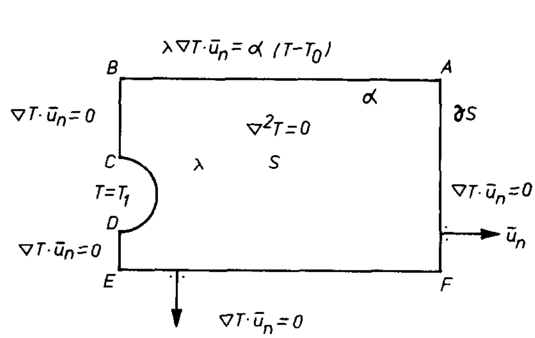

Fig. 2. Half period of the structure shown in Fig. 1.

function ρ, the boundary conditions (1)–(3) are imposed on (4). This gives:

$$\oint_{\partial S} \rho(\mathbf{r}')G(\mathbf{r}|\mathbf{r}')\mathrm{d}c' = T_1 \qquad \mathbf{r} \in CD$$

$$-\frac{\rho(\mathbf{r})}{2} + \oint_{\partial S} \rho(\mathbf{r}')\nabla G(\mathbf{r}|\mathbf{r}')\cdot\mathbf{u}_n\,\mathrm{d}c' = 0$$

$$\mathbf{r} \in BC, DE, EF \text{ and } FA$$

$$\lambda\left[-\frac{\rho(\mathbf{r})}{2} + \oint_{\partial S} \rho(\mathbf{r}')\nabla G(\mathbf{r}|\mathbf{r}')\cdot\mathbf{u}_n\,\mathrm{d}c'\right]$$

$$= \alpha\left[\oint_{\partial S} \rho(\mathbf{r}')G(\mathbf{r}|\mathbf{r}')\mathrm{d}c' - T_0\right] \qquad \mathbf{r} \in AB. \quad (7)$$

Equation (7) is an integral equation in the unknown function ρ. Once ρ is determined by solving (7) numerically, the temperature $T(\mathbf{r})$ and hence all thermal fluxes can be found by similar integrations as (4). As integral equation methods have been used in various applications [1–4], further details will be omitted here.

For the numerical solution of (7), the integrals appearing in the equation should be discretised in a suitable way. The integral equation (7) is then replaced by an algebraic set as relations like (4) are rewritten by suitable summations. For more details one is referred to the literature.

3. RESULTS

The integral equation (7) has been solved for some typical dimensions $a = 15$ cm and $d = 5$ cm. The temperature distribution along AB was calculated for several values of α/λ (Fig. 3). Taking some typical values $\alpha = 8\ \mathrm{W/m^2\ °C}$ and $\lambda = 1.4\mathrm{W/m°C}$ (for concrete), one gets $\alpha/\lambda = 5.7\ \mathrm{m^{-1}}$. Hence values of 1–100 $\mathrm{m^{-1}}$ were taken for α/λ in order to cover all possible situations. In the graphs shown on Fig. 3 it is clear

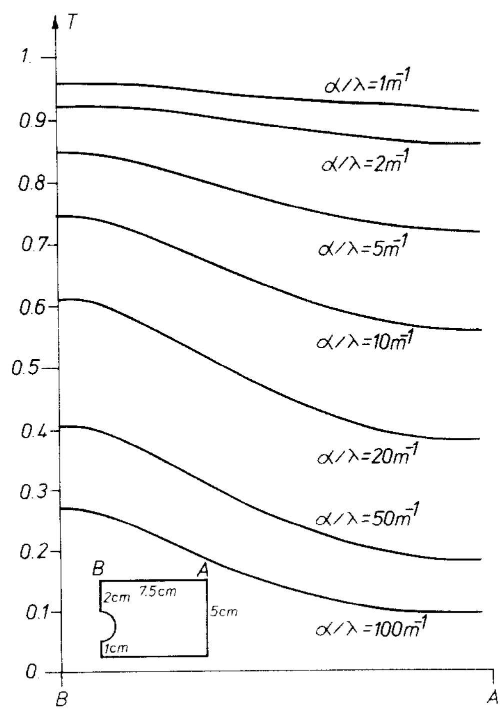

FIG. 3. Temperature variation at floor level.

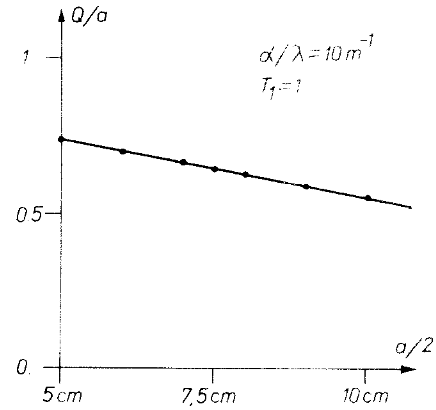

FIG. 4. Heat gain as a function of the distance a.

that α/λ has a strong influence. One should bear in mind, however, that in practical situations α/λ is almost constant, so that this conclusion has no practical application. It is more interesting to note that the temperature distribution along AB is almost constant even when the diameter in the pipes (2 cm) is much smaller than the distance a between them. Roughly speaking, Fig. 3 shows that $\pm 70\%$ of the temperature drop from T_1 to T_0 occurs in the floor for usual values of α and λ.

Figure 4 shows the heat flux per unit length Q/a as a function of the distance a. For this graph α/λ was taken to be equal to $10\ \mathrm{m^{-1}}$. When a varies from 10 to 20 cm, the results of Fig. 4 prove that Q/a is almost constant. Hence, one can conclude that for floor heating installations, the distance between the pipes is not very critical.

The influence of the floor thickness d is shown on Fig. 5 for two values of a. It is noted that the thickness d has a minor influence. By changing d from 5.5 to 7.5 cm (which means that the depth of the pipes varies from 2.5 to 4.5 cm) the variation of the heat flux is less than 10%.

Figure 6 shows the temperature distribution inside the floor. It clearly shows how the thermal energy is conducted from the heating pipe towards the ground level.

4. CONCLUSION

A numerical method to calculate the temperature distribution in a floor heating system has been presented. The method is based on an integral equation technique which enables us to apply the numerical technique to arbitrary geometries. From the results obtained in a specific case it was found that neither the floor thickness nor the distance between the heating pipes are very critical.

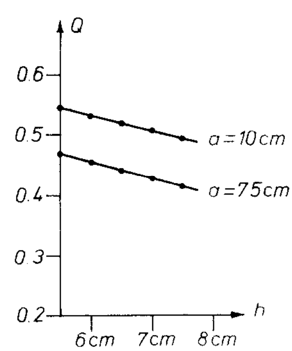

FIG. 5. Heat gain as a function of the height h.

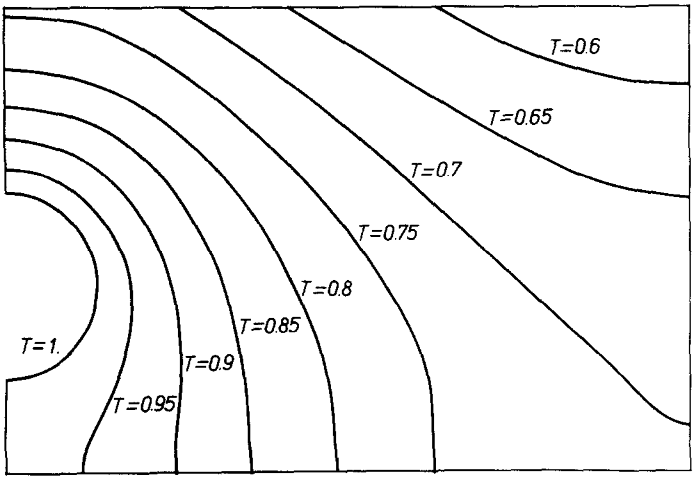

FIG. 6. Temperature distribution in the floor.

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AN ANALYTIC SOLUTION OF THE FILM THICKNESS OF LAMINAR FILM CONDENSATION ON INCLINED PIPES

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NOMENCLATURE

r, z, ψ, coordinates of the cylindrical system [m, m, −];
R, pipe radius [m];
D, pipe diameter [m];
H, pipe length [m];
θ_0, angle between pipe axis and gravity directions;
δ, film thickness [m];
v_z, v_ψ, velocity in z, and ψ direction, respectively [m s^{-1}];
g, acceleration of free fall [m s^{-2}];
ρ, density of fluid [kg m^{-3}];
η, viscosity [N s m^{-2}];
λ, thermal conductivity [Wm^{-1} K^{-1}];
L, latent heat of condensation [J kg^{-1}];
ΔT, difference between pipe wall and vapour temperature [K];
$\dot{m}$, mass flow rate [kg s^{-1}];
h, h_m, local and mean coefficient of heat transfer [Wm^{-2} K^{-1}];
C_1, C_2, constants;
ξ, Φ, dimensionless variables, equation (6);
τ, variable of integration.

1. INTRODUCTION

A DETAILED description of the laminar film condensation of pure saturated vapour on inclined cylinders has been given by Hassan and Jakob [1]. Applying Nusselt's classical theory of film condensation [2], they derived a partial differential equation for the film thickness. The numerical solution of this equation obtained by a finite difference method, serves as a basis for their further conclusions.

Reconsidering the problem of laminar film condensation on inclined pipes, the present author found that instead of a numerical one, an analytic solution of the partial differential equation by means of the method of characteristics can be given. This interesting result, not found in literature, will be described in this note. Some of the results of Hassan and Jakob [1] will be verified using the analytic expression of the film thickness.

2. THE BOUNDARY VALUE PROBLEM FOR THE FILM THICKNESS

Referring to Hassan and Jakob [1] for a more complete